Compute Taylor series for the functions

- $\cosh x = \dfrac{e^x + e^{-x}}{2}$

- $2\sin x \cos x$

- $x \ln(1 - x^3)$

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10/9/25

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$f(0) = \frac{1+1}{2} = 1$$

$$f'(0) = \frac{1-1}{2} = 0$$

$$f''(0) = \frac{1+1}{2} = 1$$

$$\cosh x = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\cosh x = \frac{e^x}{2} + \frac{e^{-x}}{2}$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{2} + \frac{(-x)^n}{2}$$

$$2\sin x\cos x$$

$$= \sin 2x$$

$$\sin 2x$$

$$= 0 + 2\cos 0\,(x) - \frac{4\sin 0\,(x^2)}{2!} - \frac{8\cos 0\,(x^3)}{3!}$$

$$+ \cdots$$

$$= 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{n+1}}{(2n+1)!}$$

$$x\ln(1-x^3)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\Rightarrow \ln(1-x^3) = -x^3 - \frac{x^6}{2} - \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots$$

$$= -x^3\left(1 + \frac{x^3}{2} + \frac{x^6}{3} + \frac{x^9}{4} + \cdots\right)$$

$$= -x^3 \sum_{n=0}^{\infty} \frac{x^{3n}}{n+1}$$

$$x\ln(1-x^3) = -x^4 \sum_{n=0}^{\infty} \frac{x^{3n}}{n+1}$$